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Research Article

# Diffusion of Information in an Online Social Network with Limited Attention

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ABSTRACT:

This article investigates the competition for limited attention in a social network with innovation. We consider the case where each piece of information has a fitness as proxy of its quality. The higher is the quality, the higher are the chances of being transmitted. We measure the relationship between the quality of an idea and its likelihood of becoming prevalent at the system level. We find that both information overload and limited attention contribute to a degradation of the system discriminative power. When trust is incorporated into the model and the agents can decide whether or not to accept a meme, we show that both lifetime and popularity distributions have broad power-law tails indicating that only a few memes spread virally through the population reproducing perfectly the broad distributions obtained from empirical data.

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## Introduction

The problem of competition for attention has become one of the most thriving topics in the field of information diffusion. With millions of people adopting online social media as their main source of news, understanding how infor-

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mation propagates is of extreme importance since theses platforms provide the perfect breeding ground for the diffusion of low quality in formation such as misinformation and fake news.[1,2,3,4] In reality, users are exposed to a very large number of memes,[5] i.e., any piece of transmissible information, and, because of the limited attention or limited cognitive capacity,[6] they cannot consume all the information they are exposed to, and therefore, only a small fraction of them will eventually become popular while the vast majority will simple disappear. There are several cases in which such a behavior is observed. Examples of such a behaviour including the number of hashtags or URL retweet on Twitter,[7] video views on YouTube,[8] citations,[9,10,11] among many others.[12,13,14,15] Aware of this, information producers employ various mechanisms to polish the way they present their product to attract most people attention.

Traditionally, models of information diffusion are based on tools borrowed from theoretical epidemiology where susceptible agents became infected by interaction with infected agents and, in spite of their simplicity, they were able to reproduce several empirical observations.[16,17,18,19,20,21] For example, Weng,[22] Gleeson [23] and Notarmuzi [24] have shown that a very simple model of information diffusion can produce a flat-tailed distribution for the popularity of a given meme. Such a kind of behaviours was commonly observed in a variety of systems that include citations,[9,10,11] hashtags and URLs on Twitter,[7] videos on YouTube,[8] among many others.[12,13,14,15] Qiu *et.al.* proposed a diffusion model that considers the user's limited attention and the quality of the information being transmitted. The authors showed that there exists a tradeoff between discriminative power and diversity. However, in realistic conditions, the model predicts that high-quality information has little advantage over low-quality information.[25] Simultaneously, Sreenivasan *et.al.* [26] proposed a model of information cascades on feed-based networks also considering the finite attention, innovations and message diffusion. In such a case, the authors estimated the branching factor associated with the cascade process for different attention spans and different forwarding probabilities. They demonstrated that beyond a certain attention span, cascades tend to become viral. Ciampaglia *et.al.* [27] proposed a model in which memes are selected based on their popularity or quality and the authors found that popularity bias hinders average quality when users are capable of exploring many items, as well as when they only consider very few top items due to scarce attention. They also found that an intermediate regime exists in which some popularity bias is good in distinguishing high-quality information, but too much can harm the system. More recently, Oliveira *et.al.* investigated the impact of influential nodes on the spreading of information. The authors showed that meme's quality does not guarantee virility, but there is a strong correlation between the meme's success and the influence of the agent who introduced it.[28] When considering situations where agents with heterogeneous criteria of quality. Cisneros-Velarde and coworkers proposed a simple method for enhancing the spread of high-quality information. Their results consist on strategically resorting the information feeds of users that share low-quality information. Under different settings of types of users, the authors showed that this policy has the best performance on homogeneous agents with a good criterion of what constitutes "good information." Moreover, they found that even in the case where agents are either purely malicious or have an opposite criterion of what constitutes high-quality information, the policy greatly reduces the spread of low quality information.[29] Although several works have been done trying to address to the crucial importance for the problem of competition for attention, there still a lack of a better understanding of how memes behave in on-line social network from the moment they are introduced into the system and start to compete for the user's attention until they are completely forgotten.

## The Model and Numerical Results

We consider an agent-based model inspired by the long tradition of representing the spread of ideas as an epidemic process where information is passed along the edges of a network.[30,31,32,33,34] The model consists of a network where each agent is equipped with a memory containing $\alpha$ memes organized in a "first-in-first-out" manner. Additionally, every meme in the system has a numerical value drawn from an uniform distribution at the moment of its creation representing its fitness or quality. Furthermore, new memes are continuously introduced into the system in an exogenous way and the rate at which this happens ultimately determines the amount of diversity in the system in the sense that the higher information load $\mu$, the harsher the competition. We assume that at time $t = t_0$ the system is in its state of higher diversity where each node has $\alpha$ unique memes. At every time step a node $i$ is selected at random and with probability $\mu$ it introduces a new meme in the system by adding it to the top of its memory and sharing it with all its neighbors. On the other hand, with probability $1-\mu$ the selected node chooses a meme from its memory and, than, transmits it to all its neighbors. Once all neighbors receive the meme, it is placed at the top of their memory, and as a consequence, the last meme in each node's list is removed or forgotten (see Figure 1 for details). Additionally, the probability of been selected is given by

$$P_i(k) = \frac{f(m_k)}{\sum_{j=1}^{\alpha} f_i(m_j)}.$$

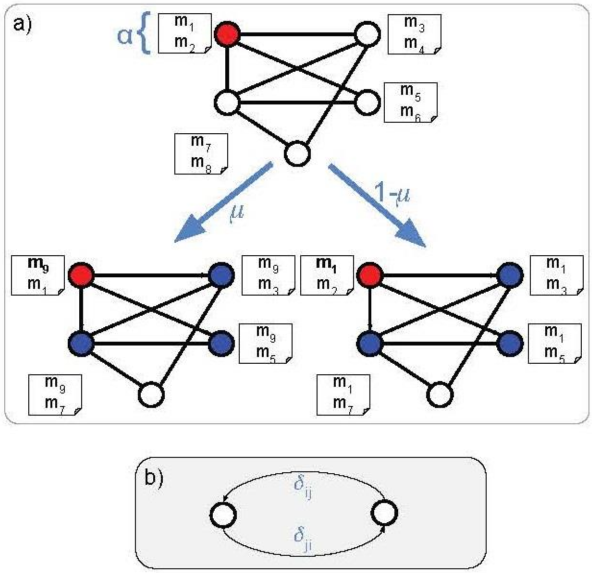


**Figure 1: Illustration of the Meme Diffusion Model.**
(a) At each time step, an agent $i$ is considered (red). The agent chooses to create a new meme with probability $\mu$ and transmit it to all its neighbors. Otherwise, with probability $1-\mu$, the agent copies a meme from its memory and transmit it to all its neighbors. The probability of selecting a meme is proportional to its fitness. (b) Introduction of trust in to the model. The parameter $\delta_{ij}$ represents the trust agent $i$ has on agent $j$.

The copying mechanism represents the adoption of a meme shared by a connection, as is done, e.g. through tweets on Twitter, shares on Facebook. It is worth to mention $\mu=1$ corresponds the case where at every time step a new meme is introduced. On the other hand, for the case of $\mu=0$, there is no innovation, and as $t\to\infty$ the number of memes alive tends to one (usually, a meme with very high quality). The proposed model allows us to study the process behind the competition for limited attention, how the information load and the quality of information affect the chances of a meme to succeed and stay on the network for long times. We start by considering the behavior of the diversity, $\bar{D}$, as a function of time for different network sizes, different values of the information load $\mu$ and different values for the attention $\alpha$. At time $t=t_0$ the system is in a state of higher diversity with $N\times\alpha$ different memes, where $N$ is the network size and as the competition starts to take place the system converges to a steady state that highly depends on $\mu$, $\alpha$ and the network size.

To investigate how the system changes from high to low diversity due to the competition, we measure the average system diversity $\overline{D}$ of an ensemble of initial conditions. First, we evaluate the average over the time for a single realization and then over an ensemble of initial condition. Thus, we have

$$\overline{D} = \frac{1}{Z}\sum_{i=1}^{Z}\frac{1}{n+1}\sum_{t=0}^{n} D_{i,t},$$

where the index $i$ corresponds to a sample of an ensemble of realization and $Z$ denotes the number of different realizations and $D_{i,t}$ is the number of different memes in the system for the sample $i$ at time $t$ as shown in Figure 2(a). We see from Figure 2(a) three different kinds of behaviors. For short $t$, the diversity remains approximately constant. Then, after a first crossover, it decays as a power law and, after a second crossover, $\overline{D}$ finally bends to a regime of constant plateau for sufficient $t$. We notice the larger is the network, the longer it takes to reach the regime of decay and then to the saturation regime. The changeover from the initial plateau to the decay and from decaying to saturation are marked by a crossover time $t_x$ corresponding to the intersection between the lines of the constant plateau and the decaying power law for the first crossover or the intersection between the lines of the decaying power law and the saturation for the second crossover. It must be emphasized that different values of the parameters $\mu$, $\alpha$ and $N$ generate different behaviors $\overline{D}$. Furthermore, the behavior shown in Figure 2 is typical from systems exhibiting scaling properties.[35,36,37,38] In this work, we will investigate the dynamics of the systems at the steady state. Once the system reaches such a state, we performed measurements to determine the success of a meme following them from the moment they were first shared until they completely disappeared from the network, recording their quality, popularity, lifetime. Here, we define popularity as the number of times a given meme is selected to be transmitted and lifetime is the time passed between the meme's creation and its extinction. The results presented here were obtained by using scale-free networks [39] with 1000 nodes and average degree 20. During each simulation, we monitored 1,000,000 memes that were introduced and forgotten after the system reached the steady state. We ran each simulation 4 times, so that our analyses took 4,000,000 memes into consideration for each combination of the parameters studied.

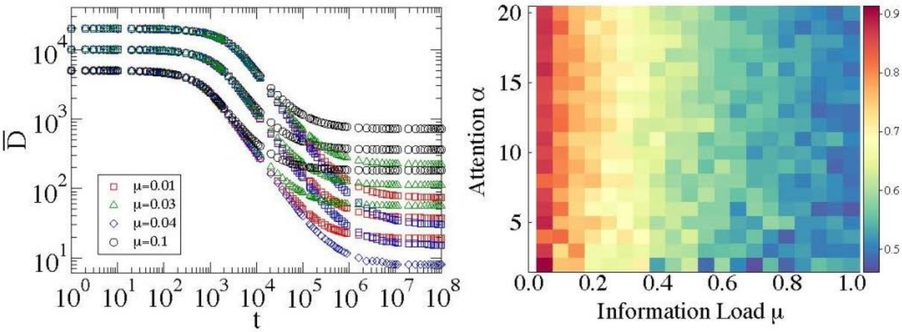


**Figure 2: (a) Behavior of the average system's diversity as a function of time for different attention and information load $\mu$. (b) Behavior of the average system's quality as a function of the attention $\alpha$ and information load $\mu$.**

The dependence of the average system quality on the attention α and information load $\mu$ is shown in Figure 2(b). We observe that by increasing the information load, users have less time to distinguish between low and high quality information in their attention list, and as a consequence the average system's quality decreases. The situation in which $\mu = 1$ represents the case in which a new meme is introduced every time step and $Q \rightarrow 0.5$ since all memes are drawn at random from an uniform distribution between 0 and 1. Next, we investigate the behavior of two measures of meme's success, namely, popularity and lifetime. The distribution of meme popularity, shown in Figure 3(a), highly depends on the information load $\mu$. For high values $\mu$, the distribution is exponentially narrow and no memes go viral. As the information load becomes lighter ( $\mu < 0.2$ ), our model reproduces the broad distribution from the empirical data, indicating that a few memes spread virally through the population. In the absence of quality, we would expect a power-law distribution of popularity $P(p) \approx p^{-1.5}$. However, fitting reveals a larger exponent $\beta \approx 1.94$ which is consistent with a model of a branching process with uniform fitness, which predicts an exponent $\beta \approx 2$. The behavior of the meme's lifetime is shown in Figure 3(b). Observe that the lifetime exhibits a peak that corresponds to the average time needed for the memes that are not selected to disappear from the network and, as the information load decreases, more long-lived viral memes appear in the network.

Furthermore, in a system in which people rely to consume information about daily events, two quantities are desirable, namely, diversity of information and discriminative power. Diversity of information, because we want to expose the

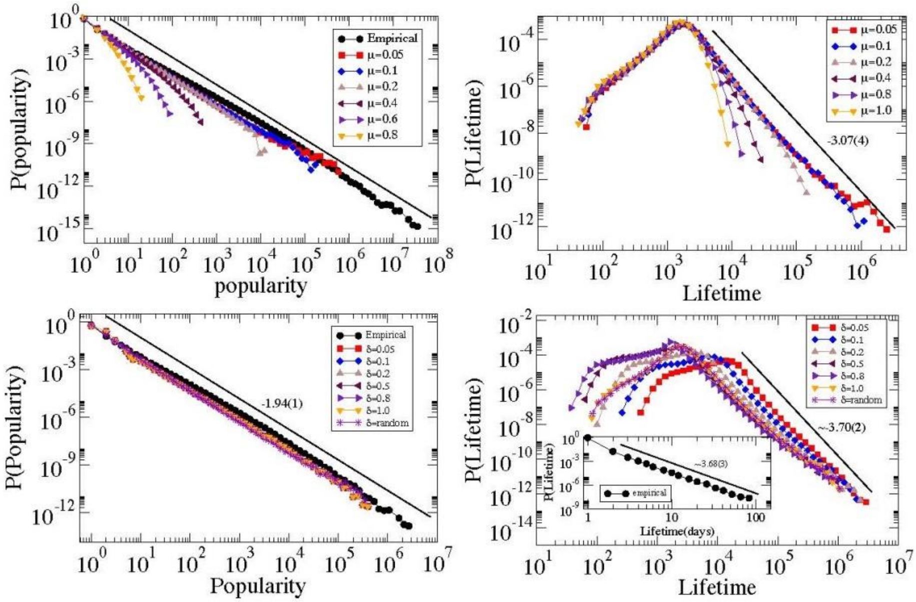


**Figure 3:** (a) Distribution (probability density function) of (a)-(c) meme's popularity and (b)-(d) meme's lifetime for different values of $\mu$ and $\delta$. We compared the model predictions with an empirical distribution obtained by counting the number of occurrences of hashtags and their lifetime from a sample of public tweets. This empirical popularity and lifetime were distributed according to a power-law distribution.

users to a wide range of point of views. At the same time, it is desirable to have a system that is able to distinguish between low- and high-quality information, or information and misinformation, or even real news and fake news. To measure the amount of diversity in the system at the steady state, we use the entropy $H = -\sum_m P(m) \log P(m)$, where $P(m)$ is the portion of attention received by meme $m$, i.e., the fraction of messages with $m$ across all of the user feeds. The sum runs over all memes present at a given time and is averaged over a long period after stationarity has been achieved. The minimum entropy is zero, when all nodes have the same meme ($\mu = 0$). However, the maximum entropy will depend on the control parameters. To discount this dependence, we measure diversity using the normalized entropy $H / H(\mu = 1)$. Figure 4(a) shows that with this normalization, the diversity does not depend in a significant way on the attention $\alpha$. However, the diversity increases with information load and is maximized for high $\mu$.

Finally, we can summarize the dependency between the quality of memes and their success in a single discriminative power measure by looking at the correlation between quality and popularity. Since the two quantities are not normally distributed, we measure the normalized mutual information (NMI) between the two variables. High normalized mutual information ($NMI \approx 1$) indicates that fitter memes are more likely to win, granting the system discriminative power; in the extreme case where $NMI = 1$, the two rankings are completely concordant. Small NMI ($NMI \approx 0$) signifies a lack of quality discrimination by the network. Figure 4(b) shows that network discriminative power degrades both with higher information load and with more limited attention.

So far, we have assumed that every user will adopt any kind of information received from their peers. However, in real life situations there are several characteristics that must be taken into account. Among them is the trust between two users. To this end, we extend the original model by introducing a parameter $\delta_{ij}$ representing the trust a user $i$ has on user $j$ (see Figure 1(b) for details). While in the original model the information transmitted by a given agent will be received by all its immediate neighbors and placed on the top of their attention list, in the new version of the model, the neighbors can decide whether or not to accept a meme based on their trust. At every time step, a number $\psi \in [0,1]$ is drawn at random and only users with $\psi \geq \delta$ will pay attention to the memes they were exposed to. Here, we consider two situations, namely (a) $\delta_{ij} = \delta_{ji} = \delta$ and (b) $\delta_{ij} \neq \delta_{ji}$. In case (a), we assume that the trust among users is a constant $\delta$, while in case (b) the trust is assigned at random. In both cases, $\delta_{ij}$ is assigned at the begging of the simulations and it does not change over time.

The distribution of meme's popularity and lifetime for different values of $\delta$ are shown in Figure 3(c-d). While both distributions have broad power-law tails indicating that only a few memes spread virally through the population. Surprisingly, the introduction of trust does not seem to have a significant effect on the popularity distribution, however, the lifetime exhibits a peak that corresponds to the average time needed for the memes that are not selected to disappear from the network and, as one can see, the lower is the trust between users, the longer it takes. Observe that, our model reproduces perfectly the broad distributions for both popularity and lifetime obtained from empirical data from Twitter.[1] Finally, Figure 4(a) shows that network discriminative power decreases as the information load increases. However, the diversity does not depend in a significant way on the trust $\delta$ as shown in Figure 4(b).

[1] The empirical distribution for the popularity was obtained by counting the number of time 1.000.000 hashtags selected at random were shared in one year. The lifetime distribution was obtained by looking at the number of consecutive days a hashtag was mentioned. In order to measure the lifetime, we removed all the hashtags that were already in the system during the first and the last two weeks of the year.

## Conclusions

In this paper, we considered a very simple agent-based model to study the diffusion of information in an on-line social network. The model allowed us to study the competition among memes in the presence of limited attention and innovation. We considered the behavior of the diversity of memes as a function of time and we showed that the dynamics of the system highly depend on the information load $\mu$. In the sense that the higher the information load, the lower the overall system quality. When trust was incorporated into the model, we showed that two of the most common metrics of success, namely, popularity and lifetime showed a broad power-law distribution indicating that only a few memes spread virally through the population, while the vast majority will simple die soon after their creation. Both distributions reproduced perfectly the broad distributions obtained from empirical data.

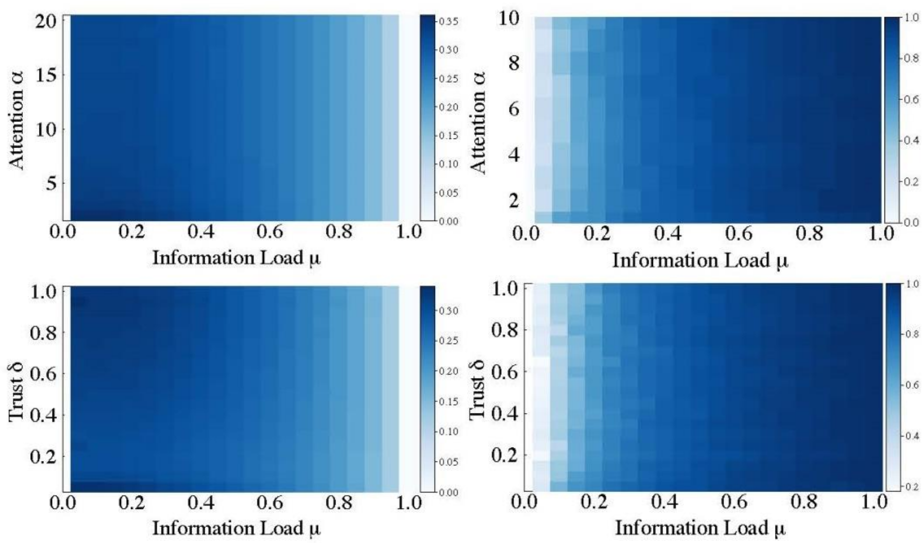


**Figure 4:** (a) Discriminative power (color scale bar) as a function of information load and finite attention. (b), Diversity $H/H(\mu=1)$ (color scale bar) as a function of intensity of information load and attention. Phase diagram for the system's (c) discriminative power (color scale bar) and (d) diversity $H/H(\mu=1)$ (color scale bar) as a function of information load $\mu$ and the trust $\delta_{ij}=\delta_{ji}=\delta$.

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## References

[1] Craig Silverman, "This Analysis Shows How Viral Fake Election News Stories Outperformed Real News on Facebook," *BuzzFeed News*, November 16, 2016, https://www.buzzfeednews.com/article/craigsilverman/viral-fake-election-news-outperformed-real-news-on-facebook.

[2] Marcella Tambuscio, Diego F.M. Oliveira, Giovanni Luca Ciampaglia, and Giancarlo Ruffo, "Network Segregation in a Model of Misinformation and Fact Checking," *arXiv preprint*, arXiv:1610.04170, https://arxiv.org/abs/1610.04170.

[3] Michela Del Vicario, Alessandro Bessi, Fabiana Zollo, Fabio Petroni, Antonio Scala, Guido Caldarelli, H. Eugene Stanley, and Walter Quattrociocchia, "The Spreading of Misinformation Online," *Proceedings of the National Academy of Sciences* 113, no. 3 (January 2016): 554–559, doi: 10.1073/pnas.1517441113.

[4] Chengcheng Shao, Pik-Mai Hui, Lei Wang, Xinwen Jiang, Alessandro Flammini, Filippo Menczer, Giovanni Luca Ciampaglia, "Anatomy of an Online Misinformation Network," *PloS One* 13, no. 4 (April 2018), doi: 10.1371/journal.pone.0196087.

[5] Richard Dawkins, *The Selfish Gene*, no. 199 (Oxford University Press, 2006).

[6] R. I. M. Dunbar, "Neocortex Size as a Constraint on Group Size in Primates," *Journal of Human Evolution* 22, no. 6 (June 1992): 469–493, https://doi.org/10.1016/0047-2484(92)90081-J.

[7] Kristina Lerman, Rumi Ghosh, and Tawan Surachawala, "Social Contagion: An Empirical Study of Information Spread on Digg and Twitter Follower Graphs," *arXiv preprint*, arXiv:1202.3162 (February 2012), http://arxiv.org/abs/1202.3162.

[8] Riley Crane and Didier Sornettette, "Robust Dynamic Classes Revealed by Measuring the Response Function of Social System," *Proceedings of the National Academy of Sciences* 105, no. 41 (October 2008): 15649–15653, http://dx.doi.org/10.1073/pnas.0803685105.

[9] Michael J. Stringer, Marta Sales-Pardo, and L. A. Nunes Amaral, "Statistical Validation of a Global Model for the Distribution of the Ultimate Number of Citations Accrued by Papers Published in a Scientific Journal," *Journal of the American Society for Information Science and Technology* 61, no. 32 (2010): 1377–1385, https://doi.org/10.1002/asi.21335.

[10] Dashun Wang, Chaoming Song, and Albert László Barabási, "Quantifying Long-term Scientific Impact," *Science* 342, no. 6154 (2013): 127–132.

[11] Orion Penner, Raj K. Pan, Alexander M. Petersen, Kimmo Kaski, and Santo Fortunato, "On the Predictability of Future Impact in Science," *Nature: Scientific Reports* 3, Article number 3052 (2013), https://www.nature.com/articles/srep03052.

[12] Sinan Aral and Dylan Walker, "Creating Social Contagion Through Viral Product Design: A Randomized Trial of Peer Influence in Networks," *Management Science* 57, no. 9 (2011): 1623–1639.

[13] Everett M. Rogers, *Diffusion of Innovations* (New York: Simon and Schuster, 2010).

[14] Salman Jamali, "Comment Mining, Popularity Prediction, and Social Network Analysis," Ph.D. Thesis (Fairfax, VA: George Mason University, 2010).

[15] Jure Leskovec, Lada A. Adamic, and Bernardo A. Huberman, "The Dynamics of Viral Marketing," *ACM Transactions on the Web* (TWEB) 1, no. 1 (2007): 5.

[16] Mark Newman, *Networks: An Introduction* (Oxford University Press, 2010).

[17] David Easley and Jon Kleinberg, *Networks, Crowds, and Markets: Reasoning About a Highly Connected World* (Cambridge University Press, July 2010).

[18] William Goffman and Vaun A. Newill, "Generalization of Epidemic Theory: An Application to the Transmission of Ideas," *Nature* 204, no. 4955 (1964): 225–228.

[19] D.J. Daley and D.G. Kendall, "Epidemics and Rumours," *Nature* 204, no. 4963 (1964): 1118.

[20] Romualdo Pastor-Satorras and Alessandro Vespignani, "Epidemic Spreading in Scale-free Networks," *Physical Review Letters* 86, no. 14 (2001): 3200-3203.

[21] Johan Ugander, Lars Backstrom, Cameron Marlow, and Jon Kleinberg, "Structural Diversity in Social Contagion," *Proceedings of the National Academy of Sciences* 109, no. 16 (April 2012): 5962–5966.

[22] Lilian Weng, Alessandro Flammini, Alessandro Vespignani, and Filippo Menczer, "Competition among Memes in a World with Limited Attention," *Nature Scientific Reports* 2, Article number: 335 (2012), https://lilianweng.github.io/files/meme.pdf.

[23] James P. Gleeson, Jonathan A. Ward, Kevin P. O'Sullivan, and William T. Lee, "Competition-Induced Criticality in a Model of Meme Popularity," *Physical Review Letters* 112, no. 4 (January 2014), 048701.

[24] Daniele Notarmuzi and Claudio Castellano, "Analytical Study of Quality-Biased Competition Dynamics for Memes in Social Media," *arXiv preprint*, arXiv:1803.08511 (2018).

[25] Xiaoyan Qiu, Diego F. M. Oliveira, Alireza Sahami Shirazi, Alessandro Flammini, and Filippo Menczer, "Limited Individual Attention and Online Virality of Low-quality Information," *Nature Human Behaviour* 1, no. 7 (2017): 1-22, doi:10.103841562-017-0132.

[26] Sameet Sreenivasan, Kevin S. Chan, Ananthram Swami, Gyorgy Korniss, and Boleslaw K. Szymanski, "Information Cascades in Feed-based Networks of Users with Limited Attention," *IEEE Transactions on Network Science and Engineering* 4, no. 2 (April-June, 2017): 120-128, https://doi.org/10.1109/TNSE.2016.262580.

[27] Giovanni L. Ciampaglia, Azadeh Nematzadeh, Filippo Menczer, and Alessandro Flammini, "How Algorithmic Popularity Bias Hinders or Promotes Quality," *Scientific reports* 8, no. 1, Article number: 15951 (2018).

[28] Diego F.M. Oliveira and Kevin S. Chan, "The Effects of Trust and Influence on the Spreading of Low and High Quality Information," *Physica A: Statistical Mechanics and its Applications* 525, Issue C (2019): 657–663.

[29] Pedro Cisneros-Velarde, Diego F.M. Oliveira, and Kevin S. Chan, "Spread and Control of Misinformation with Heterogeneous Agents," in *Complex Networks X* (Proceedings of the 10th Conference on Complex Networks CompleNet 2019), ed. Sean P. Cornelius, Clara Granell Martorell, Jesús Gómez-Gardeñes, and Bruno Gonçalves (Cham: Springer, 2019), 475-83.

[30] Brian Karrer and Mark E. J. Newman, "Competing Epidemics on Complex Networks," *Physical Review E* 84 (2011), 036106.

[31] Kim Sneppen, Ala Trusina, Mogens H. Jensen, and Stefan Bornholdt, "A Minimal Model for Multiple Epidemics and Immunity Spreading," *PloS One* 5, no. 10 (October 2010), https://doi.org/10.1371/journal.pone.0013326.

[32] Stephen Morris, "Contagion," *The Review of Economic Studies* 67, no. 1 (2000): 57–78, http://dx.doi.org/10.1111/1467-937X.00121.

[33] Norman T. Bailey, *The Mathematical Theory of Infectious Diseases and Its Applications* (High Wycombe, Bucks: Charles Grin & Company Ltd., 1975).

[34] Michaela Gotz, Jure Leskovec, Mary McGlohon, and Christos Faloutsos, "Modeling Blog Dynamics," in *Proceedings of the Third International ICWSM Conference* (2009), https://cs.stanford.edu/people/jure/pubs/blogs-icwsm09.pdf.

[35] Edson D. Leonel, Diego F. M. Oliveira, Ricardo Egydio De Carvalho, "Scaling Properties of the Regular Dynamics for a Dissipative Bouncing Ball Model, *Physica A: Statistical Mechanics and its Applications* 386, no. 1 (2007): 73–78, https://doi.org/10.1016/j.physa.2007.08.017

[36] Diego F. M. Oliveira, Rafael A. Bizao, and Edson D. Leonel, "Scaling Properties of a Hybrid Fermi-Ulam-Bouncer Model," *Mathematical Problems in Engineering* 2009 (2009): 1-14, https://doi.org/10.1155/2009/213857.

[37] Diego F. M. Oliveira, Marko Robnik, and Edson D. Leonel, "Statistical Properties of a Dissipative Kicked System: Critical Exponents and Scaling Invariance," *Physics Letters A* 376, no. 5 (2012): 723–728, https://doi.org/10.1016/j.physleta.2011.12.031.

[38] Diego F. M. Oliveira, Kevin S. Chan, Edson D. Leonel, "Scaling Invariance in a Social Network with Limited Attention and Innovation," *Physics Letters A* 382, no. 47 (2018): 3376–3380, https://doi.org/10.1016/j.physleta.2018.09.034.

[39] Albert László Barabási and Reka Albert, "Emergence of Scaling in Random Networks," *Science* 286, no. 5439 (October 1999): 509–512, https://doi.org/10.1126/science.286.5439.509.

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